Limites Remarquables

$$\frac{\ln u}{u-1} \xrightarrow{u \to 1} 1$$

Exemples :

$$\lim_{x \to +\infty} \frac{\ln(x^2-x+1)}{x} = \lim_{x \to +\infty} \frac{\ln\left(x^2\left(1-\frac{1}{x}-\frac{1}{x^2}\right)\right)}{x}$$

$$= \lim_{x \to +\infty} \frac{\ln(x^2)}{x} + \frac{\ln\left(1-\frac{1}{x}-\frac{1}{x^2}\right)}{x}$$

$$= \lim_{x \to +\infty} 2\,\frac{\ln(x)}{x} + \frac{\ln\left(1-\frac{1}{x}-\frac{1}{x^2}\right)}{x}$$

$$= 0$$

$$\lim_{x \to 0^+} \frac{\ln(x+1)}{\sqrt{x}} = \lim_{x \to 0^+} \frac{\ln(x+1)}{(x+1)-1} \times \frac{x}{\sqrt{x}}$$

$$= \lim_{x \to 0^+} \frac{\ln(x+1)}{(x+1)-1} \times \sqrt{x}$$

$$\begin{cases} \lim\limits_{x \to 0^+} x+1 = 1 \\ \lim\limits_{x \to 1} \dfrac{\ln(x)}{x-1} = 1 \end{cases}$$

$$\lim_{x \to 0^+} \frac{\ln(x+1)}{(x+1)-1} = 1$$

et $\lim\limits_{x \to 0^+} \sqrt{x} = 0$

Donc $\lim\limits_{x \to 0^+} \dfrac{\ln(x+1)}{\sqrt{x}} = 0$

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