

Calcul limite - Limite des fonctions trigonométrique - 3 -

Rappel :

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b.$$

exemple :

$$\lim_{u \rightarrow \frac{\pi}{3}} \frac{-\frac{1}{2} + \cos u}{\frac{\sqrt{3}}{2} - \sin u} = ?$$

on pose $X = u - \frac{\pi}{3} \Leftrightarrow u = X + \frac{\pi}{3}$

$$\lim_{X \rightarrow 0} \frac{-\frac{1}{2} + \cos\left(X + \frac{\pi}{3}\right)}{\frac{\sqrt{3}}{2} - \sin\left(X + \frac{\pi}{3}\right)}$$

$$\begin{aligned} \text{or } \cos\left(X + \frac{\pi}{3}\right) &= \cos X \cos \frac{\pi}{3} - \sin X \sin \frac{\pi}{3} \\ &= \frac{1}{2} \cos X - \frac{\sqrt{3}}{2} \sin X \end{aligned}$$

$$\begin{aligned} \text{et } \sin\left(X + \frac{\pi}{3}\right) &= \sin X \cos \frac{\pi}{3} + \cos X \sin \frac{\pi}{3} \\ &= \frac{1}{2} \sin X + \frac{\sqrt{3}}{2} \cos X \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{-\frac{1}{2} + \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x}{\frac{\sqrt{3}}{2} - \frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x} \\
 &= \lim_{x \rightarrow 0} \frac{-\frac{1}{2}(1 - \cos x) - \frac{\sqrt{3}}{2} \sin x}{\frac{\sqrt{3}}{2}(1 - \cos x) - \frac{1}{2} \sin x} \\
 &= \lim_{x \rightarrow 0} \frac{-\frac{1}{2} \frac{1 - \cos x}{x} - \frac{\sqrt{3}}{2} \frac{\sin x}{x}}{\frac{\sqrt{3}}{2} \frac{1 - \cos x}{x} - \frac{1}{2} \frac{\sin x}{x}} \\
 &= \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}
 \end{aligned}$$