

Calcul limite - Limite des fonctions trigonométrique - 1 -

📌 **Les formes usuelles =**

$$\lim_{x \rightarrow 0} \frac{\sin ax}{x} = a ; \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg}(ax)}{x} = a ; \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg}(x)}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(ax)}{x} = 0 ; \quad \lim_{x \rightarrow 0} \frac{1 - \cos(ax)}{x^2} = \frac{a^2}{2}$$

exemples:

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{\operatorname{tg} 2x} = \frac{\frac{\sin 7x}{x}}{\frac{\operatorname{tg} 2x}{x}} = \frac{7}{2}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{\operatorname{tg}^2 x} &= \lim_{x \rightarrow 0} \frac{\frac{1 - \cos 3x}{x^2}}{\frac{\operatorname{tg}^2 x}{x^2}} \\ &= \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{\left(\frac{\operatorname{tg} x}{x}\right)^2} \\ &= \frac{9}{2} = \frac{9}{2} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{\sin 2x} &= \lim_{x \rightarrow 0} \frac{\frac{1 - \cos 4x}{x}}{\frac{\sin 2x}{x}} \\ &= \frac{0}{2} = 0 \end{aligned}$$

$$\begin{aligned} \lim_{n \rightarrow 0} \frac{\cos(4n) - 1}{x^2} &= \lim_{u \rightarrow 0} \frac{1 - \cos 4u}{x^2} \\ &= -\frac{16}{2} = -8 \end{aligned}$$

AUTREMENT

$$\lim_{n \rightarrow 1} \frac{\sin(2(n-1))}{3(n-1)}$$

$$\lim_{n \rightarrow 1} \frac{\sin(2(n-1))}{3(n-1)} = ?$$

on pose $X = n - 1$.

$$\begin{aligned} & n \rightarrow 1 \quad \text{donc} \quad X \rightarrow 0 \\ \lim_{X \rightarrow 0} \frac{\sin 2X}{3X} &= \frac{2}{3} \end{aligned}$$

$$\begin{cases} \lim_{n \rightarrow 1} (n-1) = 0 \\ \lim_{n \rightarrow 0} \frac{1}{3} \frac{\sin 2n}{n} = \frac{2}{3} \end{cases}$$

$$\text{Ainsi } \lim_{n \rightarrow 1} \frac{\sin(2(n-1))}{3(n-1)} = \frac{2}{3}$$